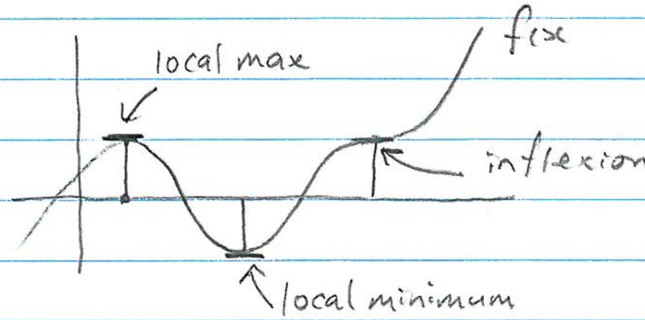
9/10/12

Lec 6

Critical Points

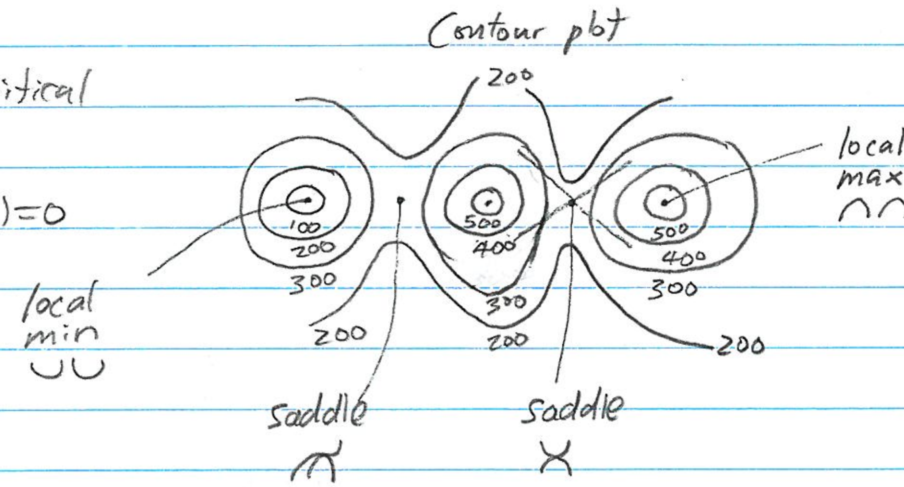
Given $f(x)$ the critical points are where $f'(x)=0$. We can distinguish them by computing $f''(x)$



min	max	inflexion
$f''(x)>0$	$f''(x)<0$	$f''(x)=0$

Given $f(x,y)$ the critical points occur when

$f_x(x,y)=0$ AND $f_y(x,y)=0$



We might guess to distinguish them using.

max	min	saddle
$f_{xx}(a,b)<0$	$f_{xx}(a,b)>0$	$f_{xx}(a,b)\cdot f_{yy}(a,b) \leq 0$
$f_{yy}(a,b)<0$	$f_{yy}(a,b)>0$	different signs.

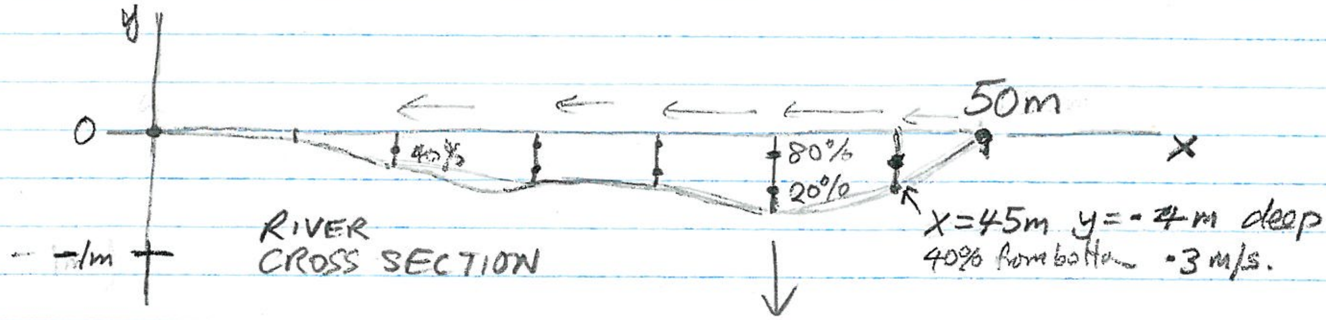
This doesn't work because if $f_{xx}(a,b)>0$ and $f_{yy}(a,b)>0$ the point (a,b) could be a saddle. See X for saddle in above plot.

Let $D(x,y) = f_{xx}f_{yy} - f_{xy}^2$. Then

$D(a,b)<0 \Rightarrow$ saddle

$D(a,b)>0 \Rightarrow$ local min or max.

How do we measure the flow rate (in m^3/s) of water coming down a river?



x=40m y=.6m
80% .5 m/s
20% .3 m/s

Averaged two = .4m/s is an estimate for the average velocity.

① flow = Area? × Average? velocity
m^2 × m/s

② velocity $v(x,y)$ = some formula??

$$\text{flow} = \int_{\text{Area}} v(x,y)$$

We don't have a formula for $v(x,y)$ near the river bed.

③ Approximate river cross-section by trapezoids?
Can we do better? Simpson's rule? What about $v(x,y)$

$v(x)$ = average velocity at x