Assignment 3, MACM 204, Fall 2014

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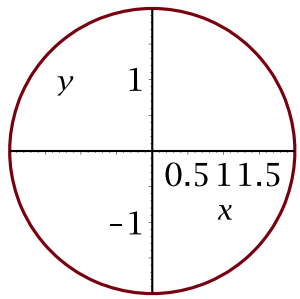
Due Tuesday October 21st at 4:00pm.
Late penalty: -20% for up to 48 hours late. 0 after that.

Please attempt each question in a seperate worksheet (so that you don't destroy your previous work). Print your Maple worksheets (you may print double sided if you wish).

Question 1

The easiest way to get a plot of a circle of radius r is to graph the equation of the circle $x^2+y^2=r^2$ using the implicitplot command like this (for $r=2$).

```
> with(plots):
  implicitplot( x^2+y^2=4, x=-2..2, y=-2..2, scaling=constrained );
```



But it's expensive to use a general curve plotting routine. A faster (and more accurate) way is to use the trigonometric parameterization of the circle which is

$x(t) = r \cdot \cos(t),\ y(t) = r \cdot \sin(t)$ for $-\pi \le t \le \pi$.

You can get a parametric plot in Maple using the plot command of $x(t)$ and $y(t)$ for $a \le t \le b$ as follows

```
plot( [x(t),y(t),t=a..b] );
```

Graph the circle for radius 2 using the style=point option so you can see that the points computed are equally spaced around the circle.

Now the equation for a sphere of radius r is $x^2+y^2+z^2=r^2$. You can graph this using the implicitplot3d command in the plots package like this

```
> with(plots):
  implicitplot3d( x^2+y^2+z^2=4, x=-2..2, y=-2..2, z=-2..2 );
```

But this is also very expensive. Find a trigonometric parameterization for the sphere of radius r using google or in a book. It will be of the form

$(x(s, t),\ y(s, t),\ z(s, t))$ for $a \le s \le b$ and $c \le t \le d$

i.e. there are two parameters s and t.
Graph it for radius 2 using the plot3d command as follows

```
plot3d( [x(s,t), y(s,t), z(s,t)], s=a..b, t=c..d );
```

Question 2

Solve the following linear systems in Maple using the solve command.

```
> sys1 := [x+y+z=2, x-y=1, x+z=0];
```

$$sys1 := [x + y + z = 2, x - y = 1, x + z = 0]$$

```
> sys2 := [x+y+z=2, x-y=1, 2*y+z=1];
```

$$sys2 := [x + y + z = 2, x - y = 1, 2\, y + z = 1]$$

```
> sys3 := [x+y+z=2, x-y=1, 2*y+z=-1];
```

$$sys3 := [x + y + z = 2, x - y = 1, 2\, y + z = -1]$$

Recall that an equation of the form $a \cdot x + b \cdot y + c \cdot z = d$ is the equation of a plane in 3 dimensions. Explain Maple's answers visually by graphing each system (of three planes) on the same plot using the implicitplot3d command for a suitable domain. Give the three planes different colours. A one sentence explanation is sufficient.

Question 3

Consider the function

```
> f := 4*y^3+x^2-12*y^2-36*y+x^3+2;
```

$$4\, y^3 + x^2 - 12\, y^2 - 36\, y + x^3 + 2$$

Find the critical points using Maple. You should get 4 of them. Determine which are saddle points, which are local minimums and which are local maximums.

Graph the surface $f(x, y)$ and highlight the critical points by drawing a spike (vertical line) through them using the **spacecurve** command in the plots package. Use blue for the saddles and red for the others so we can tell which is which.

Question 4

Without exectuting the following Maple program, explain in one sentence, what it does.

```
  g := proc(M::list,n::nonnegint) local f;
         if n=0 then return 0 fi;
         if M[n]<50 then f := 1; else f := 0; fi;
         f + g(M,n-1);
     end;
```

Question 5

$g(a, x) = a^2 \cdot x \cdot (1 - x) \cdot (a \cdot x^2 - a \cdot x + 1)$. We'd like to determine the solutions of the two polynomial equations $g(a, x) = x$ and $g_x(a, x) = -1$. By $g_x(a, x)$ I mean the partial derivative $\frac{\partial}{\partial x}\, g(a, x)$.

Part (a). Graph the curves $g(a, x) = x$ and $g_x(a, x) = -1$ in different colors using the implicitplot command on the domain $1 \le a \le 4$ and $0 \le x \le 1$ so you can see where the solutions are. You should see two solutions, one near $(x = 0.8,\ a = 3.5)$ and the other near $(x = 0.4,\ a = 3.5)$.

Part (b). Use the `fsolve` command with appropriate options to solve for the solutions.

Part (c). Use the `solve` command to solve for the solutions. There are lots of solutions, some are complex. Find and simplify the two solutions that we want. Note the solve command will return **implicit** formulas with RootOfs in them. To get **explicit** formulas for the solutions from solve first execute this command

```
> _EnvExplicit := true;
```

Question 6

Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and vectors $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Part (a) Input A, u and v into Maple and calculate $2 \cdot u + w$, A^3 and A^{-1} .

Part (b) Using commands in the **LinearAlgebra** package calculate the determinant of *A*, the characteristic polynomial of *A* in the variable x and solve the linear systems $A \cdot x = u$ and $A \cdot x = w$. Note, to load the package and see all the commands in the package do

```
> ?LinearAlgebra
```

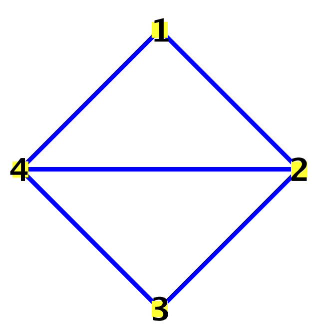
Part (c) Starting with the vector $v_1 = [1, 0]$ compute, in a for loop, the vectors

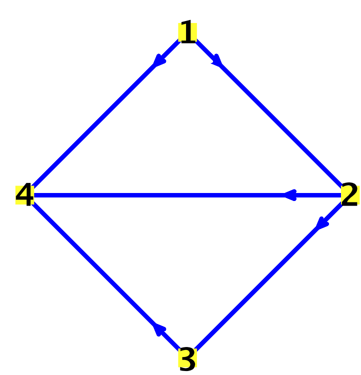
$$v_2 = A \cdot v_1,\ v_3 = A \cdot v_2,\ v_4 = A \cdot v_3,\ \ldots,\ v_{10} = A \cdot v_9$$

What numbers appear in the sequence of vectors $v_1, v_2, v_3, \ldots, v_{10}$?

Question 7

Part (a) input the following graphs shown below to Maple using the Graph command in the GraphTheory package and draw them using the DrawGraph command. The first is an undirected graph (no direction on the edges), the second is a directed graph (see the little arrows on the edges).





Part (b) For both graphs, the neighbors of vertex 3 are vertices 2 and 4. Read the help page for the GraphTheory package and find a command in the GraphTheory package that computes the neighbors and execute it on both graphs.

```
> ?GraphTheory
```

Question 8

Below is data for the velocity of a river that represents measurements taken at different positions accross the river.

The data value $[x, y, v]$ means at position *x* meters the river is *y* meters deep and we measured the velocity of the water at position *x* at depth 40% above the river bed to be *v* meter per second. You can use v as the average velocity at position *x*.

The data value $[x, y, v_1, v_2]$ means at position *x* meters the river is *y* meters deep and we have measured the velocity to be v_1 $m \cdot s^{-1}$ 20% up from the river bed and v_2 $m \cdot s^{-1}$ 20% from the surface of the river. The average of v_1 and v_2 would give an estimate for the average velocity at position x.

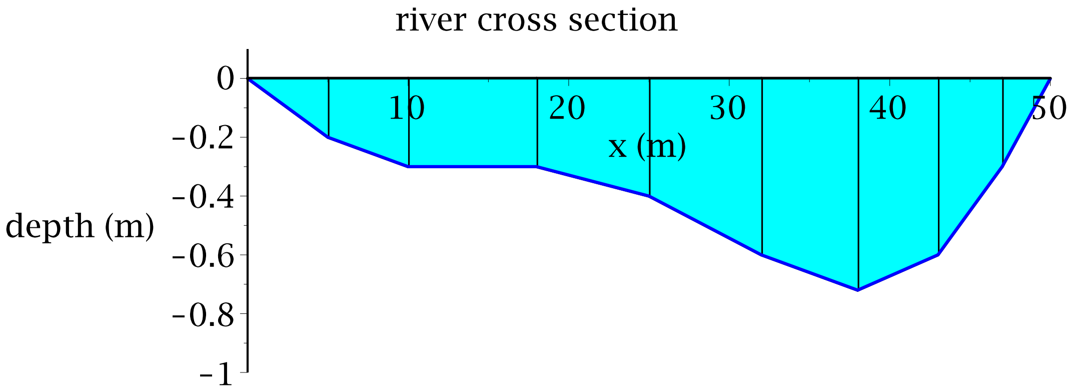
You are to estimate the total flow of the river in $m^3 \cdot s^{-1}$. Your Maple code should work for any data of this form. You may assume the first data point is for one side of the river and the last data point is the other side of the river and that there are at least 3 data points, so at least one measurement was taken.

In class I suggested that we approximate the flow on each section $x_L \le x \le x_R$ with depths d_L and d_R and velocities v_L and v_R by approximating the area of that section by a trapezoid and the velocity on that section by the average of the velocities v_L and v_R on either side of the section. Compute the flow using this approach. This will underestimate the flow when $d_R > d_L$ and $v_R > v_L$. For a bonus (of 2% of final grade) find a more accurate formula for the flow. Compute the flow using your improved formula to determine whether it makes much difference on this data.

```
> Data :=
  [ [0,0.0,0.0],
    [5,0.2,0.1],
    [10,0.25,0.2],
    [18,0.3,0.3],
    [25,0.4,0.3,0.50],
    [32,0.6,0.35,0.55],
    [38,0.72,0.40,0.60],
    [43,0.6,0.30,0.60],
    [47,0.3,0.30],
    [50,0.0,0.0] ];
```

Question 9

For the river flow data in question 8, generate a plot of the cross section of the river. For each section of the river i.e. for $xL \le x \le xR$ with depths dL and dR generate a blue trapezoid and assemble the plot so that it looks like the figure below. See the polygon command in the plottools package.



Question 10

Consider a random walk in the XY plane where at each time step you walk one step (one unit) either to the left, right, up or down, at random. Starting from the origin, generate plots for at least two random walks with at least n=1000 random steps (n=10,000 is much better).

So first create a list of n values P := $[[0, 0], [x_1, y_1], [x_2, y_2],..., [x_n, y_n]]$.
You can also use an array of points here here instead of a Maple list.
Then you can simply graph them using the plot(P, style=line); command.
To get random numbers from 1,2,3,4 use the following

```
> R := rand(1..4):
```

Now when you call R() you will get one of 1,2,3,4 at random, e.g.,

```
> R(), R(), R();
```

$$3, 3, 2$$