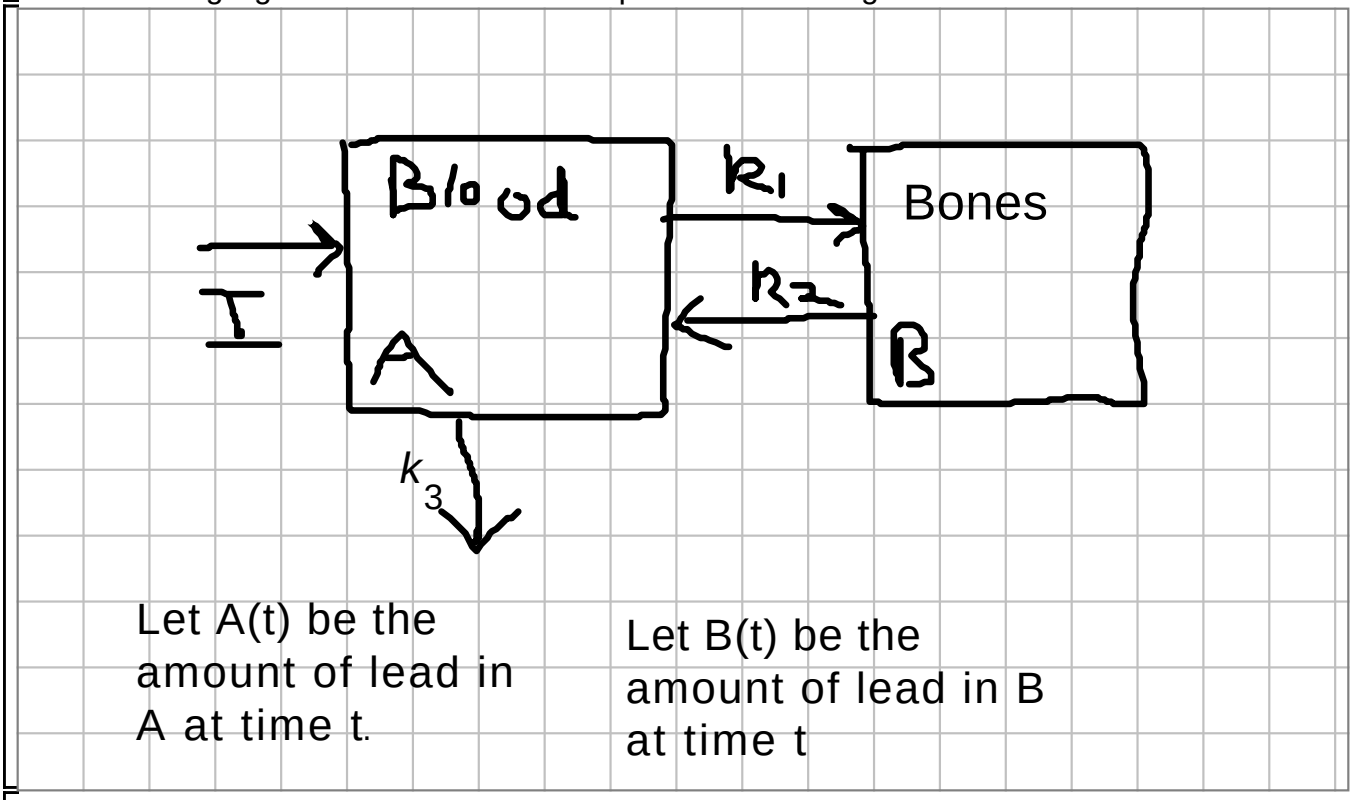
The Lead model

A(t) is the amount of lead in the blood at time t
B(t) is the amount of lead in the bones at time t
J is the amount of lead going into the body from the environment (assume constant)

The following figure was created in Maple: see Drawing under the Insert menu.




```
> eq1 := diff(A(t),t) = J -k1*A(t) - k3*A(t) + k2*B(t);
  eq2 := diff(B(t),t) = k1*A(t) - k2*B(t);
```

$$eq1 := \frac{d}{dt} A(t) = J - k1\, A(t) - k3\, A(t) + k2\, B(t)$$

$$eq2 := \frac{d}{dt} B(t) = k1\, A(t) - k2\, B(t)$$

Equilibrium occurs when $diff(A(t), t) = 0$ **and** $diff(B(t), t) = 0$.

```
> sys := { rhs(eq1) = 0, rhs(eq2) = 0 };
```

$$sys := \{k1\, A(t) - k2\, B(t) = 0,\ J - k1\, A(t) - k3\, A(t) + k2\, B(t) = 0\}$$

```
> LeadEquil := solve( sys, {A(t),B(t)} );
```

$$LeadEquil := \left\{A(t) = \frac{J}{k3},\ B(t) = \frac{J\, k1}{k2\, k3}\right\}$$

```
> sol := dsolve( {eq1,A(0)=0,eq2,B(0)=0}, {A(t),B(t)} ):
```

UGLY FORMULA not shown

```
> J := 1;
  k1 := 0.1;
  k2 := 0.02;
```

```
> k3 := 0.1;
```

$$J := 1$$
$$k1 := 0.1$$
$$k2 := 0.02$$
$$k3 := 0.1$$

```
> LeadEquil;
```

$$\{A(t) = 10.00000000,\ B(t) = 50.00000000\}$$

```
> sol;
```

$$\{A(t) = -5.497518590\, e^{-0.009501243800\, t} - 4.502481400\, e^{-0.2104987562\, t} + 10.00000000,$$
$$B(t) = 2.363522729\, e^{-0.2104987562\, t} - 52.36352270\, e^{-0.009501243800\, t} + 50.00000000\}$$

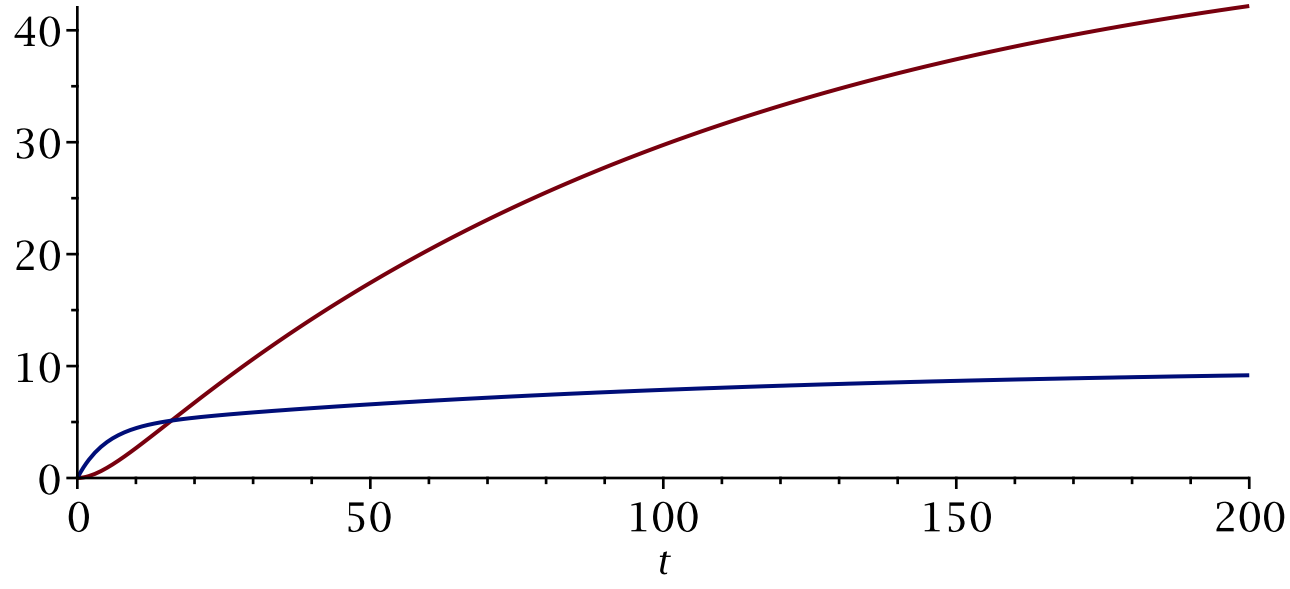
```
> map( F, {a,b,c} );
```

$$\{F(a), F(b), F(c)\}$$

```
> map( rhs, sol );
```

$$\{2.363522729\, e^{-0.2104987562\, t} - 52.36352270\, e^{-0.009501243800\, t} + 50.00000000,$$
$$-5.497518590\, e^{-0.009501243800\, t} - 4.502481400\, e^{-0.2104987562\, t} + 10.00000000\}$$

```
> plot( map(rhs,sol), t=0..200 );
```




```
> map( F, {a,b,c}, x );
```

$$\{F(a, x), F(b, x), F(c, x)\}$$

```
> map( limit, {A(t),B(t),C(t)}, t=infinity );
```

$$\{\lim_{t\to\infty} A(t), \lim_{t\to\infty} B(t), \lim_{t\to\infty} C(t)\}$$

```
> map(limit, sol, t=infinity );
```

$$\{\lim_{t\to\infty} A(t) = 10., \lim_{t\to\infty} B(t) = 50.\}$$

```
> LeadEquil;
```

$$\{A(t) = 10.00000000,\ B(t) = 50.00000000\}$$

```
> with(DEtools):
```

```
> DEplot( {eq1,eq2}, {A(t),B(t)}, t=0..300,
  {[A(0)=0,B(0)=0],[A(0)=10,B(0)=0],[A(0)=10,B(0)=80]},
  linecolor=blue, numpoints=200, arrows=medium );
```