MATH 801/CMPT 891, Assignment 6, Spring 2021.

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Due 11pm Friday April 15th. Late Penalty: −20% for up to 48 hours late. Zero after that.

Question 1: Rational Function Integration (20 marks)

Reference: sections 11.3, & 11.4 and 11.5.

(a) Calculate the Trager-Rothstein resultant for the rational function integral below by hand by constructing Sylvester's matrix for the resultant and computing the determinant by hand. Complete the integral.

$$\int \frac{2x+1}{x^2-2}\mathrm{dx}.$$

(b) Integrate the three rational functions below as follows. First compute the rational function part of integral using either Hermite's method or Horrowitz's method (or both if you wish). Then compute the logarithmic part (if any) using the Trager-Rothstein resultant. Use Maple to help with arithmetic e.g. you may use `gcd`, `gcdex`, `solve`, `resultant` etc.

$$f_1 = \frac{3\,x^5 - 2\,x^4 - x^3 + 2\,x^2 - 2\,x + 2}{x^6 - x^5 + x^4 - x^3}$$

$$f_2 = \frac{4\,x^7 - 16\,x^6 + 28\,x^5 - 351\,x^3 + 588\,x^2 - 738}{2\,x^7 - 8\,x^6 + 14\,x^5 - 40\,x^4 + 82\,x^3 - 76\,x^2 + 120\,x - 144}$$

$$f_3 = \frac{6\,x^5 - 4\,x^4 - 32\,x^3 + 12\,x^2 + 34\,x - 24}{x^6 - 8\,x^4 + 17\,x^2 - 8}$$

Question 2: Non-elementary Integrals (20 marks)

Reference: 12.6, 12.7

Apply the Risch algorithm to prove that the following integrals are not elementary.

$$\int \frac{1}{\log(x)^2}\mathrm{d}x$$

$$\int \frac{\log(x)}{x+1}\mathrm{d}x$$

$$\int \frac{e^x}{x}\mathrm{d}x$$

$$\int e^x \log(x)\mathrm{d}x$$

Question 3: Elementary Integrals (20 marks)

Reference: 12.6, 12.7

Apply the Risch algorithm to compute the following elementary integrals.

$$\int 2\theta + 2 - \frac{1/x + 1}{(\theta + x)^2} + \frac{1}{x\theta}\ \mathrm{d}x \qquad \text{where} \ \ \theta = \log(x)$$

$$\int \theta + x\theta + \frac{2}{x}\theta^2 - \frac{1}{x^2}\theta^2\ \mathrm{d}x \qquad \text{where} \ \ \theta = e^x$$

$$\int \frac{e^{2x}}{e^{2x} + e^x + 1}\ \mathrm{d}x$$

$$\int \ln x + e^x \ln x + \frac{e^x}{x^2}\ \mathrm{d}x$$

Read the proof of Theorem 12.3 (Differentiation of exponential polynomials) on page 520.