Lec22A The Logarithmic Part Michael Monagan

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Ch. 11 Rational Function Integration.

Th 11.6 Let $P, Q \in K[x]$, K a field, $\deg P < \deg Q$, $\gcd(P,Q) = 1$, $\operatorname{lc} Q = 1$. There exist $A, B, C, D \in K[x]$ with $\deg A < \deg B$ and $\deg C < \deg D$ s.t.

$$\int \frac{P}{Q} = \frac{A}{B} + \int \frac{C}{D} \quad \text{where } \overset{\text{monic}}{B} = \gcd(Q, Q') \text{ and } \overset{\text{monic}}{D} = Q/B.$$

$\frac{C}{D}$ ← square-free.

$B = q_2 q_3^2 \cdots q_k^{k-1}$ $\qquad D = q_1 q_2 \cdots q_k$.

$\frac{A}{B}$: rational part — Hermite, Horrowitz

$\int \frac{C}{D}$: logarithmic part. — Trager-Rothstein.

If $Q = q_1 q_2^2 \cdots q_k^k$ with $\gcd(q_i, q_j) = 0$ and $\gcd(q_i, q_i') = 1$.

$$\Rightarrow \boxed{\gcd(D, D') = 1.}$$

E.g. $\int \frac{dx}{x^3 + x}$, $x^3 + x = x(x^2+1)$ $\Rightarrow$ $A = 0$, $C = 1$, $B = 1$, $D = x^3 + x$.

Idea:
$$\frac{C}{D} = \frac{C}{(x-\beta_1)(x-\beta_2)\cdots(x-\beta_n)} = \frac{\alpha_1}{x-\beta_1} + \frac{\alpha_2}{x-\beta_2} + \cdots + \frac{\alpha_n}{x-\beta_n}$$

$K = \cancel{\mathbb{Q}}$ $\quad \beta_i \in \mathbb{C}$ $\quad \alpha_i \in \mathbb{C}$.

$$\Rightarrow \int \frac{C}{D} = \alpha_1 \ln(x-\beta_1) + \cdots + \alpha_n \ln(x-\beta_n).$$

If $\alpha_i = \alpha_j$
$$= \cdots\cdots \alpha_i \ln(x-\beta_i) + \cdots + \alpha_i \ln(x-\beta_j) + \cdots$$
$$= \alpha_i \ln(\underbrace{(x-\beta_i)\cdot(x-\beta_j)}_{v_i}).$$

Can we compute α_i, v_i without computing β_i roots of $D(x)$?
Yes see Theorem 11.7 Trager Rothstein (first handout).