

An integral domain E is a Euclidean domain if

$\exists v: E \setminus \{0\} \rightarrow \mathbb{N} \cup \{0\}$ satisfying

- (i) $v(ab) \geq v(a) \quad \forall a, b \in E \setminus \{0\}$
- (ii) $\forall a, b \in E, b \neq 0 \quad \exists q, r \in E$ satisfying
 $a = bq + r$ with $r = 0$ or $v(r) < v(b)$

Properties of E

Let u be a unit in E and c, d be non-zero non-units in E

- (iii) $v(u) = v(1)$
- (iv) $v(c) > v(1)$
- (v) $v(uc) = v(c)$
- (vi) $v(cd) > v(c)$
- (vii) $c|d$ and $d|c \Rightarrow v(c) = v(d)$
- (viii) $v(d) > v(c) \Rightarrow d \nmid c$