MATH 240 Assignment 4, Spring 2016

Please put your name and student ID at the top of the front page and staple your assignment together.
Please hand in to the dropoff boxes outside AQ 4135 by 6pm Wednesday March 9th.
Sorry, no late assignments are accepted.
A reminder that quiz 3 is on Friday March 11th at the beginning of class.

Michael Monagan

4.7 Exercises 2, 8, 13.

3.1 Exercises 1, 10, 30.

3.2 Exercises 5, 22, 32, 33, 34, 35, 40.

3.3 Exercises 4, 20, 29.

For exercises 32, 33, 34, 35 and 40 of 3.2, use Theorems 3,4,5,6.
For exercises 20 and 29 of 3.3 use determinants.

Additional exercises on Polynomial Interpolation.
For these four exercises, see my notes which I will post on Canvas. In the exercises we want to find the quadratic polynomial $f(x)$ that interpolates the 3 points $(x_1, y_1) = (1, 1), (x_2, y_2) = (0, -1)$ and $(x_3, y_3) = (2, 9)$, i.e., $f(x_1) = y_1, f(x_2) = y_2$ and $f(x_3) = y_3$. We use three different bases for P_2 to do this, the standard basis $B = \{1, x, x^2\}$, the Newton basis and the Lagrange basis.

1 Let $V_3 = \begin{bmatrix} 1 & x_1 & {x_1}^2 \\ 1 & x_2 & {x_2}^2 \\ 1 & x_3 & {x_3}^2 \end{bmatrix}$. Show that $\det V = (x_1 - x_2)(x_1 - x_3)(x_2 - x_3)$.

2 Find the polynomial $f(x) = Ax^2 + Bx + C$ interpolating the 3 data points above by solving the linear system $V_3 z = y$ for z where V_3 is the 3×3 matrix above and $z = [C, B, A]$ and check that $f(x)$ interpolates the points.

3 For the 3 points above the Newton basis is $N = \{1, (x-1), (x-1)x\}$. We found in class that $f(x) = 1 + 2(x-1) + 3(x-1)x$. Thus the co-ordinate vector $[f]_N = [1, 2, 3]$. We also found that $[f]_B = [-1, -1, 3]$ where $B = \{1, x, x^2\}$ is the standard basis. Find the change of basis matrix P from N to B. Check that $P \times [f]_N = [f]_B$.

4 For $n = 3$ points the Lagrange basis

$$L = \{(x - x_2)(x - x_3), (x - x_1)(x - x_3), (x - x_1)(x - x_2)\}.$$

Show that if $x_1 \neq x_2 \neq x_3$ then L is a linearly independent set of polynomials. Hint: show that $c_1 L_1(x) + c_2 L_2(x) + c_3 L_3(x) = 0$ has only the trivial solution $c_1 = c_2 = c_3 = 0$.