Merging and Sparse Polynomial Addition

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Assignment #4 posted. Due next Tuesday @ 11pm.

Computing with Multivariate Polynomials.

Sparse polynomial addition using merging.
In $\mathbb{Z}[x]$ consider $h = f + g$. e.g.

$$f = 7x^{10} - 3x^8 + 6x^3 + 4$$
$$+\ g = 3x^8 + 6x^3 + 5x^2$$
$$= \quad h = 7x^{10} + 12x^3 + 5x^2 + 4.$$

Assume terms are sorted by degree in x in descending order.
Assume a sparse representation.

$f = [\,[7,10],\ [-3,8],\ [6,3],\ [4,0]\,]$ ($7x^{10}$, $-3x^8$, $6x^3$, $4\cdot x^0$)

$g = [\,[3,8],\ [6,3],\ [5,2]\,]$

$h = [\,[7,10],\ [12,3],\ [5,2],\ [4,0]\,]$

$i = 1 \to 2 \to 3 \to 4$ (STOP); $j = 1 \to 2 \to 3 \to 4$ (STOP); $k = 0 \to 1 \to 2 \to 3 \to 4$

$h_k = f_i$ copy; $h_k \leftarrow [12,3]$

Compare f_i with g_j

$x^{10} > x^8$

$x^8 = x^8$

$3 + -3 = 0.$

$x^3 = x^3$

$6 + 6 = 12.$

$x^0 < x^2$

How many comparisons x^{i}_{inf} with x^{j}_{ing} did we do?
Let #f be the number of terms in f.
After every comparison we advance i or j or both.
The worst case is when we never advance both i and j
#f + #g − 1 comparisons. Which is linear.

$x^1 \cdot (3x^2 + 5x + 2) = 3x^3 + 3x^2 + 2x$

How can we multiply f × g.
Let $f = \sum_{i=1}^{m} f_i$ where $m = \#f$ and $g = \sum_{i=1}^{n} g_i$ with $n = \#g$.

(ii)

$$f \times g = (\cdots(((f_1 \cdot g + f_2 g) + f_3 \cdot g)\cdots) + f_m \cdot g = \sum_{i=1}^{n} f_i \cdot g.$$

merge merge. merge.