FFGE

September 26, 2023 9:05 AM

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    |\^/|     Maple 2021 (X86 64 LINUX)
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 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> n := 5;
                                   n := 5

> A := Matrix(n,n,rand(10^4));
                      [7926    8057       5    3002    2347]
                      [                                    ]
                      [9765    3354    5860    6906    5281]
                      [                                    ]
                 A := [5393    1203     311    9386    9810]
                      [                                    ]
                      [5144    7995    3121    9390    2055]
                      [                                    ]
                      [6505    5293    2987    2440    8012]

> det := LinearAlgebra[Determinant](A);
                       det := 23791466233143137296
```

20 digits.

```
> mu := 1:
> for k to n-1 do
>     mu := mu*A[k,k]^(n-k-1);
>     for i from k+1 to n do
>         for j from k+1 to n do
>             A[i,j] := A[k,k]*A[i,j]-A[i,k]*A[k,j];
>         od;
>         A[i,k] := 0;
>     od;
>     print(A[k+1..n,k+1..n]);
>     printf("max length = %d digits\n",length(max(seq(seq(abs(A[i,j]),j=k+1..n),i=k+1..n))));
> od:
                 [-52092801    46397535    25422426    18938751]
                 [                                             ]
                 [-33916423     2438021    58203650    65096689]
                 [                                             ]
                 [ 21923162    24711326    58982852     4214962]
                 [                                             ]
                 [-10458467    23642437     -188570    48235877]

max length = 8 digits
          [ 1446635080430484    -2169753403021452    -2748734175828216]
          [                                                            ]
          [-2304462863969796    -3629921935279464     -634766482939224]
          [                                                            ]
          [ -746353677117192     275702742865512    -2314671639266760]

max length = 16 digits
     [-10251288552034411837957632844368 , -7252631293222107209343437468352]
     [                                                                     ]
     [-12205621711824463499652169349 76 ,  -5400013052587912787382804201312]

max length = 32 digits
        [46504804588789939837283646317990315771751006790507796595531264]

max length = 62 digits
> det = A[n,n]/mu;
                23791466233143137296 = 23791466233143137296
```

Let $A^{(0)} = A$ and $A^{(k)}$ be the matrix after the k'th step

Ordinary Gaussian elimination

$$R_i \leftarrow R_i - \frac{A_{ik}^{(k-1)}}{A_{kk}^{(k-1)}} \cdot R_k$$

for $k = 1$ to $n-1$ (step k)

for $i = k+1$ to n (row i)

for $j = k+1$ to n

$$A_{ij}^{(k)} := A_{ij}^{(k-1)} - \frac{A_{ik}^{(k-1)}}{A_{kk}^{(k-1)}} \cdot A_{kj}^{(k-1)}$$

$$\begin{array}{c|c} & \begin{matrix} \ \ & k & j \end{matrix} \\ \begin{matrix} \\ k \\ \\ i \end{matrix} & \begin{bmatrix} 4 & \times & \times \\ 0 & \boxed{3} & 2 \\ 0 & \times & \times \\ 0 & \boxed{5} & 11 \end{bmatrix} \end{array}$$

$$= 11 - \frac{5}{3} \cdot 2 = \frac{23}{3}$$

$$\downarrow$$

Clear fractions

$$R_i \leftarrow A_{kk}^{(k-1)} R_i - A_{ik}^{(k-1)} R_j$$

$$\det(A^{(k)}) = \left(A_{kk}^{(k-1)}\right)^{n-k} \det(A^{(k-1)})$$

$$A_{ij}^{(k)} := A_{kk}^{(k-1)} A_{ij}^{(k-1)} - A_{ik}^{(k-1)} A_{kj}^{(k-1)}$$

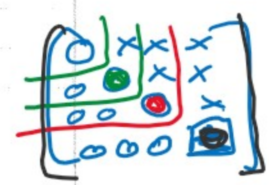
$$\downarrow$$

Bareiss/Edmonds: $A_{00}^{(-1)} := 1$

$$A_{ij}^{(k)} := \frac{A_{kk}^{(k-1)} A_{ij}^{(k-1)} - A_{ik}^{(k-1)} A_{kj}^{(k-1)}}{A_{k-2\,k-2}^{(k-2)}}$$

current pivot

previous pivot



Theorem (Jack Edmonds 1967, Erwin Bareiss 1968, Jordan 1838–1922)

The division by $A_{k-2\,k-2}^{(k-2)}$ is exact in R and $\det(A) = A_{nn}^{(n-1)}$

Moreover $A_{kk}^{(k-1)} = \det(k \times k$ principle minor of $A)$ and

$$\underset{i \geq k,\, j \geq k}{A_{ij}^{(k-1)}} = \det(k \times k \text{ minor of } A)$$