

Assignment 1 Question 1 Solutions

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Part (a)

> **int(x*(1-x), x);**

$$-\frac{1}{3}x^3 + \frac{1}{2}x^2 \quad (1)$$

> **int(x^2*exp(-x), x);**

$$-(x^2 + 2x + 2)e^{-x} \quad (2)$$

> **int(4*sqrt(1-x^2), x);**

$$2x\sqrt{-x^2+1} + 2\arcsin(x) \quad (3)$$

> **int(x*(1-x), x=0..1);**

$$\frac{1}{6} \quad (4)$$

> **int(x^2*exp(-x), x=0..infinity);**

$$\frac{2}{2} \quad (5)$$

> **int(4*sqrt(1-x^2), x=0..1);**

$$\pi \quad (6)$$

Part (b)

> **f := sin(x)+cos(x);**

$$f := \sin(x) + \cos(x) \quad (7)$$

> **eval(f,x=0); # f(0);**

$$1 \quad (8)$$

> **fp := diff(f,x); # f'(x)**

$$fp := \cos(x) - \sin(x) \quad (9)$$

> **eval(fp,x=0); # f'(0)**

$$1 \quad (10)$$

> **fpp := diff(fp,x); # f''(x)**

$$fpp := -\sin(x) - \cos(x) \quad (11)$$

> **eval(fpp,x=0); # f''(0)**

$$-1 \quad (12)$$

Part (c)

> **for k from 1 to 6 do S := sum(i^k,i=1..n); factor(S); od;**

$$S := \frac{(n+1)^2}{2} - \frac{n}{2} - \frac{1}{2}$$

$$\frac{n(n+1)}{2}$$

$$S := \frac{(n+1)^3}{3} - \frac{(n+1)^2}{2} + \frac{n}{6} + \frac{1}{6}$$

$$\frac{n(n+1)(2n+1)}{6}$$

$$\begin{aligned}
S &:= \frac{(n+1)^4}{4} - \frac{(n+1)^3}{2} + \frac{(n+1)^2}{4} \\
&\quad \frac{(n+1)^2 n^2}{4} \\
S &:= \frac{(n+1)^5}{5} - \frac{(n+1)^4}{2} + \frac{(n+1)^3}{3} - \frac{n}{30} - \frac{1}{30} \\
&\quad \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \\
S &:= \frac{(n+1)^6}{6} - \frac{(n+1)^5}{2} + \frac{5(n+1)^4}{12} - \frac{(n+1)^2}{12} \\
&\quad \frac{(n+1)^2 n^2 (2n^2+2n-1)}{12} \\
S &:= \frac{(n+1)^7}{7} - \frac{(n+1)^6}{2} + \frac{(n+1)^5}{2} - \frac{(n+1)^3}{6} + \frac{n}{42} + \frac{1}{42} \\
&\quad \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}
\end{aligned} \tag{13}$$

Part (d) and (e)

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> p := 1001;
  while not isprime(p) do p := p+2; od:
  p;
      p := 1001
      1009

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(14)

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> p := nextprime(1000);
  while irem(p-1,64) <> 0 do p := nextprime(p); od:
  p;
      p := 1009
      1153

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(15)

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> ifactor(p-1);
      (2)7 (3)2

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(16)