Assignment 2 Question 1

Part (a)

```
> sum1 := expand(sum(sum(sum(1,j=k+1..m),i=k+1..n),k=1..n));
```

$$sum1 := \frac{1}{2} n^2 m - \frac{1}{2} n m - \frac{1}{6} n^3 + \frac{1}{6} n \quad (1)$$

```
> sum2 := sum((n-k)*(m-k),k=1..n-1);
```

$$sum2 := \frac{1}{2} n^2 m - \frac{1}{2} n m - \frac{1}{6} n^3 + \frac{1}{6} n \quad (2)$$

```
> sum1-sum2;
```

$$0 \quad (3)$$

Part (b) the mixed radix representation

```
> m1,m2,m3 := 5,6,7;
```

$$m1, m2, m3 := 5, 6, 7 \quad (4)$$

```
> u1,u2,u3 := 2,3,2;
```

$$u1, u2, u3 := 2, 3, 2 \quad (5)$$

```
> u := v1+v2*m1+v3*m1*m2;
```

$$u := v1 + 5\, v2 + 30\, v3 \quad (6)$$

```
> u1 = u mod m1;
```

$$2 = v1 \quad (7)$$

```
> v1 := 2;
```

$$v1 := 2 \quad (8)$$

```
> u2 = u mod m2;
```

$$3 = 2 + 5\, v2 \quad (9)$$

```
> v2 := solve(u2=u mod m2,v2) mod m2;
```

$$v2 := 5 \quad (10)$$

```
> u3 = u mod m3;
```

$$2 = 6 + 2\, v3 \quad (11)$$

```
> v3 := solve(u3 = u mod m3,v3) mod m3;
```

$$v3 := 5 \quad (12)$$

```
> u;
```

$$177 \quad (13)$$

```
> chrem([u1,u2,u3],[m1,m2,m3]);
```

$$177 \quad (14)$$

Part (c) the Lagrange representation for u = v1 m1 m2 + v2 m1 m3 + v3 m1 m2.

```
> v1,v2,v3 := 'v1,v2,v3';
```

$$v1, v2, v3 := v1, v2, v3 \quad (15)$$

```
> u := v1*m2*m3 + v2*m1*m3 + v3*m1*m2;
```

$$u := 42\, v1 + 35\, v2 + 30\, v3 \quad (16)$$

```
> u1 = u mod m1;
```

(17)

$$2 = 2\,v1 \quad (17)$$

```
> v1 := 1;
```

$$v1 := 1 \quad (18)$$

```
> u2 = u mod m2;
```

$$3 = 5\,v2 \quad (19)$$

```
> v2 := 3/5 mod m2;
```

$$v2 := 3 \quad (20)$$

```
> u3 = u mod m3;
```

$$2 = 2\,v3 \quad (21)$$

```
> v3 := 1;
```

$$v3 := 1 \quad (22)$$

```
> u;
```

$$177 \quad (23)$$

Part (d)
The largest value of u = v1 m2 m3 + v2 m1 m3 + v3 m1 m2
then v1 = m1-1 and v2 = m2-1 and v3 = m3-1 would give the largest value of u.

```
> v1,v2,v3 := m1-1, m2-1, m3-1;
```

$$v1, v2, v3 := 4, 5, 6 \quad (24)$$

```
> u;
```

$$523 \quad (25)$$

We need to work backwards to deterine u1,u2,u3. If u = 523 then

```
> u1 := u mod m1;
  u2 := u mod m2;
  u3 := u mod m3;
```

$$u1 := 3$$
$$u2 := 1$$
$$u3 := 5 \quad (26)$$

Let's double check by redoing the Lagrange method for these values of u1,u2,u3

```
> v1,v2,v3 := 'v1','v2','v3';
```

$$v1, v2, v3 := v1, v2, v3 \quad (27)$$

```
> u1 = u mod m1;
```

$$3 = 2\,v1 \quad (28)$$

```
> v1 = 3/2 mod m1;
```

$$v1 = 4 \quad (29)$$

```
> u2 = u mod m2;
```

$$1 = 5\,v2 \quad (30)$$

```
> v2 = 1/5 mod m2;
```

$$v2 = 5 \quad (31)$$

```
> u3 = u mod m3;
```

$$5 = 2\,v3 \quad (32)$$

```
> v3 = 5/2 mod m3;
```

$$v3 = 6 \tag{33}$$