A3Q1

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The code allocates two arrays of size $n/2$ hence

$$S(n) = 2S\left(\frac{n}{2}\right) + 2\cdot\frac{n}{2} \quad \text{with } S(1)=0. \quad \text{Let } n=2^k$$

$$\left.\begin{aligned}
&\Rightarrow S(n) = \cancel{2S\left(\tfrac{n}{2}\right)} + n \\
&\Rightarrow \cancel{2S(n/2)} = \cancel{4S\left(\tfrac{n}{4}\right)} + 2\tfrac{n}{2} \\
&\qquad \vdots \\
&\Rightarrow \cancel{\tfrac{n}{2}S(2)} = \cancel{n\cdot S(1)} + \tfrac{n}{2}\cdot 2 \\
&\quad \cancel{nS(1)} = 0
\end{aligned}\right\} \text{This is } k \text{ lots of } n \text{ so } S(n) = n\log_2 n.$$

The recurrence for $A(n)$ is

$$\left.\begin{aligned}
A(n) &= \cancel{2A\left(\tfrac{n}{2}\right)} + 2 \\
\cancel{2A\left(\tfrac{n}{2}\right)} &= \cancel{4A\left(\tfrac{n}{4}\right)} + 4 \\
&\vdots \\
\cancel{\tfrac{n}{2}A(2)} &= \cancel{nA(1)} + n \\
\cancel{nA(1)} &= 0.
\end{aligned}\right\} \begin{aligned}&\text{This is } 2+4+\cdots+2^k = 2^{k+1}-2 = 2n-2. \\ &\text{So } A(n) = 2n-2.\end{aligned}$$