A3Q2

October 30, 2023 9:21 PM

(a) We are given ω is a p.n.r.u. so $\omega^n = 1$ and $\omega^i \neq 1$ for $0 < i < n$.
Also $n = 2m$. So $(\omega^2)^m = \omega^{2m} = \omega^n = 1$.
Suppose ω^2 is not a p.m.r.u. Then $\exists i$ s.t. $(\omega^2)^i = 1$ for $0 < i < m$.
$\Rightarrow \omega^{2i} = 1$. But $2i < n \Rightarrow \omega$ is not a p.n.r.u. ⊠.

(b). (i). The Fourier transform of $a(x) = x^n + c$ is

$$[a(\omega^i) : 0 \leq i < n] = [(\omega^i)^n + c = (\omega^n)^i + c : 0 \leq i < c]$$
$$= [1 + c : 0 \leq i < n].$$

(ii) For $a(x) = 1 + x + x^2 + \cdots + x^{n-1}$
Now $a(\omega^0) = a(1) = n$.
Note that $\frac{1 - x^n}{1 - x} = 1 + x + x^2 + \cdots + x^{n-1} = a(x)$
So for $i \neq 0$, $a(\omega^i) = \frac{1 - (\omega^i)^n}{1 - \omega^i} = \frac{1 - (\omega^n)^i}{1 - \omega^i} = 0$.
So the Fourier transform is $[n, 0, 0, \cdots, 0]$.