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       |\^/|        Maple 2019 (X86 64 LINUX)
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 <____ ____>   Waterloo Maple Inc.
      |        Type ? for help.
> with(LinearAlgebra):
> Berkowitz := proc(A::Matrix,x)
> local n,r,i,j,k,Ar,Cr,R,S,Q,Cn;
>     n := RowDimension(A);
>     if n=1 then return A[1,1]-x fi;
>     r := n-1;
>     Ar := A[1..r,1..r];
>     Cr := Berkowitz(Ar,x);
>     R := A[n,1..r];
>     S := A[1..r,n];
>     # compute Q[i] = R^T Ar^i S for 0 <= i < r-1
>     Q[0] := R.S; # r mults
>     for i to r-1 do S := Ar.S; Q[i] := R.S; od; # (r-1)(r^2+r) mults
>     Cn := Cr*(A[n,n]-x) # r + binomial(r,2) mults
>         + add( add( coeff(Cr,x,k+j)*Q[j]*x^(k-1), j=0..r-k ), k=1..r );
>     expand(Cn);
> end:

> r := n-1;
```
$$r := n - 1$$

```
> R := rsolve( {T(n) = T(r)+ r+(r-1)*(r^2+r)+ r+binomial(r,2), T(1)=0}, T(n) ):
> T(n) = expand(R);
```
$$T(n) = -1/3\ n^3 + 1/4\ n^2 - 1/6\ n + 1/4\ n^4$$

```
> A := Matrix([[1,2,3,4],[5,7,8,1],[8,0,1,1],[9,8,5,1]]);
```
$$A := \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 7 & 8 & 1 \\ 8 & 0 & 1 & 1 \\ 9 & 8 & 5 & 1 \end{bmatrix}$$

```
> Berkowitz(A,x);
```
$$x^4 - 10\ x^3 - 59\ x^2 - 23\ x - 937$$

```
> CharacteristicPolynomial(A,x);
```
$$x^4 - 10\ x^3 - 59\ x^2 - 23\ x - 937$$

```
> A := RandomMatrix(5,5):
> C := Berkowitz(A,x);
```
$$C := -x^5 - 24\ x^4 - 1315\ x^3 - 1309182\ x^2 - 62350034\ x - 319347140$$

```
> eval(C,x=0);
```
$$-319347140$$

```
> Determinant(A);
```
$$-319347140$$

```
> quit
memory used=8.6MB, alloc=41.3MB, time=0.12
```