

Let  $A^{(0)} = A$  and  $A^{(k)}$  be the matrix after the  $k$ 'th step

Ordinary  
Gaussian elimination

$$R_i \leftarrow R_i - \frac{A_{ik}^{(k-1)}}{A_{kk}^{(k-1)}} \cdot R_k$$

for  $k=1$  to  $n-1$  (step  $k$ )  
for  $i=k+1$  to  $n$  (row  $i$ )

$$\text{for } j=k+1 \text{ to } n \quad A_{ij}^{(k)} := A_{ij}^{(k-1)} - \frac{A_{ik}^{(k-1)}}{A_{kk}^{(k-1)}} \cdot A_{kj}^{(k-1)}$$

$$\begin{bmatrix} & k & & j \\ k & 4 & x & x \\ & 0 & 3 & x \\ & 0 & x & x \\ i & 0 & 5 & 11 \end{bmatrix}$$

$$= 11 - \frac{5}{3} \cdot 2 = \frac{23}{3}$$

Clear fractions  
 $R_i \leftarrow A_{kk}^{(k-1)} R_i - A_{ik}^{(k-1)} R_k$

$$\det(A^{(k)}) = (A_{kk}^{(k-1)})^{n-k} \det(A^{(k-1)})$$

$$A_{ij}^{(k)} := A_{kk}^{(k-1)} A_{ij}^{(k-1)} - A_{ik}^{(k-1)} A_{kj}^{(k-1)}$$

Bareiss/Edmonds:  $A_{00}^{(-1)} := 1$

$$A_{ij}^{(k)} := \frac{A_{kk}^{(k-1)} A_{ij}^{(k-1)} - A_{ik}^{(k-1)} A_{kj}^{(k-1)}}{A_{k-2k-2}^{(k-2)}}$$

$\nearrow$  current pivot  $\leftarrow$  previous pivot

Theorem (Jack Edmonds 1967, Erwin Bareiss 1968, Jordan 1838-1922)  
The division by  $A_{k-2k-2}^{(k-2)}$  is exact in  $R$  and  $\det(A) = A_{nn}^{(n-1)}$

Moreover  $A_{kk}^{(k-1)} = \det(k \times k \text{ principle minor of } A)$  and

$$A_{ij}^{(k-1)} = \det(k \times k \text{ minor of } A)$$

$i, j > k$